Dynamically Consistent Diagnostic Expectations*

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Abstract

Diagnostic expectation incorporates the psychological heuristics that people excessively inflate the probability of events which are typical or representative for the newly observed information. Under diagnostic expectation, people update their beliefs distortedly. It accounts for overreaction, systematic forecast errors, and revisions. However, the overestimated states are determined by the comparison between past rational distribution and current rational distribution. It makes current distorted beliefs irrelevant from past and future distorted beliefs because past distorted beliefs are not used to form current and future beliefs. To tackle this dynamic inconsistency and prevent multi-selves problem, this paper aims to build the dynamically consistent diagnostic expectation model. Making past distorted beliefs as a base of current distorted beliefs reveals that there is an upper bound of the strength of the psychological heuristics for diagnostic expectations. This paper also conducts simulations to verify dynamically consistent model and compare to inconsistent model with analysts' forecast data. I find that the impact of psychological heuristics is weak and analysts refer more recent beliefs in consistent model. Still, I also find that analysts significantly overreact to information even in consistent model. It is also found that consistent model has lower Euclidean distances to realized data than inconsistent model, implying that it fits more to realized data. By the consistent diagnostic model, researchers can apply diagnostic models to various fields without multi-selves problem. The results also suggest that stock market analysts exhibit overreaction tendency, but they update their beliefs based on their own distorted beliefs.

Keywords: Diagnostic expectations; dynamic consistency; overreaction; representativeness heuristics

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1 Introduction

It is widely known that many types of economic agents overreact to newly observed information. La Porta (1996) shows that stock market analysts excessively expect firms' future growth. Greenwood and Shleifer (2014) show that investors' expectations tend to be extrapolative. It should be also noted that these overreactions are followed by the predictable forecast errors. For example, Gennaioli and Shleifer (2018) show the overreaction and predictable forecast errors of stock market analysts and Bordalo et al. (2018) show that credit spreads overeact to news and entail predictable reversals.

In the rational expectation models, overreaction and subsequent predictable forecast errors are difficult to explain so that psychology-based models are proposed. One distinguished example is the "representativeness heuristics." It is the tendency that people overweight the probability of an event when it is representative of characteristics to its parent population (Kahneman and Tversky 1972).

Bordalo et al. (2018) and Bordalo et al. (2019) propose the diagnostic expectations (hereafter DE) which is based on the representativeness heuristics. DE is psychologically founded model and distinguished from mechanical models of extrapolation. There is an expanding literature on DE.

The intuition behind DE is as follows. When new information arrives, agents compare the past rational beliefs[1] and current updated beliefs. If some events are more likely to occur under current updated beliefs, agents overinflate such events. For example, if agents observed positive signal about productivity, then the likelihood of high productivity state is increased after signal. Therefore, agents overestimate the probability ofhigh productivity state whereas they underestimate the probability of low productivity state. Their ultimate beliefs are distorted.

DE succesfully explain the mechanism of overreaction and subsequent forecast errors, but it makes current distorted beliefs irrelevant from past distorted beliefs. Because past rational beliefs, not distorted beliefs, are used to form the current distot beliefs under DE, DE always

[1] Precisely, it is not beliefs at the past time, but a beliefs at current time when the observed signal is exactly same to the predicted values. In such atomic situation, no additional information is available and the expected value does not change from the past beliefs.

starts from rational beliefs which no agents actually have. In that sense, DE can be seen as an one-shot deviation from rational expectations. I call this type of DE as an one-shot model or one-shot DE.

Since one-shot DE is not dynamically consistent, multi-selves problem can arise. In such situation, the outcome evaluations would change by time (O'donoghue and Rabin 1999, O'donoghue and Rabin 2001), which makes it difficult to use revealed preferences and compare data dynamically. It would be also hard to implement welfare analysis using one-shot DE. To apply DE for various fields, dynaically consistent model is desired than one-shot model.

The purpose of this paper is to build the diagnostic expectation model which has dynamic consistency. By this model, I complement DE literature and derive some implications for agents' rationality. Bordalo et al. (2018) and Bordalo et al. (2019) assume that reference beliefs which is a base of DE are past rational beliefs. I replace it to past distorted beliefs to allow dynamic consisten for distorted agents. Then, I run the simulated method of moments (SMM) to estimate the model parameters and compare dynamic model and one-shot model.

The dynamic model in this paper involves subtle changes from one-shot model in previous studies. They use the rational beliefs as the reference distribution to determine the overestimated beliefs. Such setting ensures that agents' overreaction is always in the same direction to the rational expectations, but their past beliefs would have no effect on current beliefs and reactions. To address the latter issue, I use the distorted beliefs as the reference distribution. It allows agents' past beliefs imact on updated beliefs. Current distorted beliefs in my model is the mixed results of the subjective surprises derived from past own distorted beliefs and distorted update rule formed by the psychological heuristics. I call this model as dynamically consistent model in this paper to distinguish one-shot model.

There are several important implications derived from dynamically consistent model. Firstly, I find that the strength of the representativeness may be overestimated in dynamically inconsistent model. There are two sources to make agents overreact in DE. The one is agents' perceived surprise and the other is the representativeness. In the one-shot DE model, the magnitude of perceived surprise is not large because it is derived from past rational expectations. As a result, overreaction is only driven by the representativeness heuristics, leading the parameter of the strength high.

In contrast, in the dynamically consistent DE model, agents' past beliefs are distorted so that perceived surprise tends to be large. It is natural result from overreaction. For example, if investors observed positive signal in previous period, they overestimated the good productivity sates, but current realized signal is usually lower than expected, which makes perceived surprise large. The representativeness heuristics are not required to be strong to generate overreaction if perceived surprise is high. I derive the upper bound for the strength of the representativeness in dynamically consistent DE model.

Secondly, my simulations show that there is a significant belief distortion by representativeness even in dynamically consistent model. Simulations in this paper follow the Bordalo et al. (2019) methodology and include the rational expectation case where there are no effects of representativeness. I observe that the strength of representativeness is significantly positive, rejecting the rational expectations model.

Thirdly, I compare the consistent and one-shot DEs and find that consistent model fits better to realized data than one-shot model. Estimated strength of the representativeness in consistent model is lower than one-shot model as expected. The reference periods are also important parameters for DE. They are the lags of reference distribution used for forming DE. It is observed that consistent model refers to more recent beliefs than one-shot model. Under the consistent model, the distorted belief paths may diverge if the reference perios are not close and the perceived surprises become large.

As a robustness test, I run another simulations which fix the macroeconomic parameters to estimate the strength of representativeness and the reference periods. This setting eliminates the effects from jointly estimating the macroeconomic parameters and allows us to analyze other two parameters which are closely related to DE. Firstly, it is observed that the estimated strength in consistent model is lower than one-shot model. Secondly, I find that with sufficiently strong representativeness impacts, one-shot model fits worse to realized data, but beliefs in consistent model diverges. Thirdly, I find that consistent model has lower Euclidean distance which is used as criteria to estimate the parameters than one-shot model. It suggests that consistent model is more plausible to account for realized data than one-shot model.

The main contribution of this paper is offering the DE which has dynamic consistency. In my model, agents update their beliefs based on their own distorted beliefs and their update rule is also distorted by the representativeness heuristics. In the one-shot DE, agents update their beliefs based on past rational expectations, which enables us to understand the mechanism of agents' overreaction concisely (Bordalo et al. 2018 and Bordalo et al. 2019) but less clear the dynamic relation of agents' beliefs. This paper reveals how agents distortedly update their beliefs from distorted past beliefs. Building the dynamically consistent DE model enables us to apply DE to various fields and conduct welfare analyses. In addition, it is not obvious whether distorted updation are stationary. This paper offers the necessary conditions for consistent DE to be stationary. I also provide the evidence that one-shot model has tendency of overestimation of the strength of the representativeness.

This paper also offers new insights for the analysts' rationality. In the simulation, I use the analysts' forecasts of EPS. I find that both consistent and one-shot model can reject the null hypothesis that anaysts have rational expectations. I also find that consistent model fits better than one-shot model. These results suggests that investors are influenced by the representativeness heuristics, but their beliefs are formed based on their own past beliefs. They overreact to newly observed information, but do not forget their own beliefs. This paper sheds light on their rationality. It also suggests that the stock market investors are also equipped with dynamic consistentcy.

This paper is organized as follows. Section two summarizes the existing literature of DE. Section three describes the DE and the consistent one. Section four describes the simulation methodology and section five shows the result of baseline simulation and robustness test. Section six concludes.

2 Prior literature

If agents have rational expectations and update their beliefs by Bayes' rule, they do not over- or underreact to observing information. However, prior literature offers the evidence of agents' overreaction. For example, in the stock market, La Porta (1996) and Gennaioli and Shleifer (2018) shows the overreaction of analysts and Greenwood and Shleifer (2014) shows that many types of US investors exhibit extrapolative characteristics. d'Arienzo (2020) shows that analysts overrect in long-term bond returns. Augenblick et al. (2021) find that almost all participants update their beliefs in the right direction, but there is heterogeneity in how much they revise.

To account for these expectation formation, some studies have employed the psychological foundations such as investor sentiment and psychological heuristics. Barberis et al. (1998) propose the investor sentiment driven model and Barberis and Shleifer (2003) examine style investing where investors categorize assets into broad classes and allocate their funds. Gennaioli and Shleifer (2010) take account of representativeness heuristics, the tendency that people overweight the probability of an event when it is representative of characteristics to its parent population (Kahneman and Tversky 1972).

Based on the psychological background, Bordalo et al. (2018) propose the DE. In this framework, expectations are formed and distorted by representativeness heuristics. People overestimate the probability of event which is more likely to occur under the parent population. For example, if investor receive the positive signal, rational investors update their beliefs according to the Bayes' rule, but diagnostic investors, who has DE, overestimate the high productivity state because such state is representative to positive signals. Therefore, diagnostic investors overreact to positive signals and their expectation becomes more optimistic.

DE share some characteristics with extrapolative expectations, but there is a clear difference. Under the extrapolative expectations, agents assume that future expected changes such as returns or prices are determined by the weighted average of past changes (Barberis et al. 2018; Glaeser and Nathanson 2017; Barberis et al. 2015). Extrapolative expectations are backward-looking, whereas DE are forward-looking because they compare prior and posterior probabilities and overestimate the probability of a representative states in the posterior probability distribution. Therefore, it is a context-dependent framework because the state and the extent agents overreact depends on the prior beliefs.

DE accounts for investors' tendency of overreacting beliefs and predictable forecast errors. Bordalo et al. (2018) show that credit spreads are excessively volatile, overreact to news, and entail predictable reversals under DE. Bordalo et al. (2021b) show that when productivity growth decreases, credit spreads excessively increase due to DE. Bordalo et al. (2022) shows that overreaction due to DE generates the predictable boom-bust cycles.

Bordalo et al. (2019) apply DE to stock return analysis and show that DE can explain the analysists forecasts' systematic forecast errors and revisions. Bordalo et al. (2020b) show that systematic overreaction generates the price reversals and excess stock market volatility. Bordalo et al. (2021a) show that price overreaction leads to endogenous bubbles and crash.

DE are used to explain the consumption behavior. L'Huillier et al. (2021) show that consumption also overreacts to supply shocks. Bianchi et al. (2021a) explain the persistent and hum-shaped boom-bust cycle of consumptions. Bianchi et al. (2021b) show that consumption becomes time-inconsistent when reference point is not recent.

Krishnamurthy and Li (2020) introduce DE into frictional financial intermediation models and show that banks with DE have higher risk tolerance, decrease the risk premia and increase the credit before crisis. Maxted (2024) shows that disappointment after boom due to excessive optimism of DE make banks tighten their lending, leading to crisis.

DE establishes the distorted beliefs due to representativeness heuristics by comparing the past and current rational expectations. It makes current distorted beliefs irrelevant from past and future ones. This enables a concise explanation of overreaction to newly observed information, but does not satisfy dynamic consistency.

In the absence of dynamic consistency, multi-selves problem emerges. In this case, the outcome evaluations and prefereces would change at each time, making it difficult to use revealed prefereces. In addition, welfare analysis become hard. Therefore, there is a risk to evaluate economic agents' actions when researchers apply DE to other analyses.

This dynamic inconsistency has been analyzed in various studies so that some accomodations are desirable for applying DE. Hyperbolic discounting is one of the approaches which is characterized by the relatively high discount rates in the short run and relatively low discount rates in the long run (Laibson 1997; O'donoghue and Rabin 1999, Thaler and Shefrin 1981; Fudenberg and Levine 2012).[2] If there is a present bias effect, people wiould procrastinate if costs are immediate and preproperate if rewards are immediate (O'donoghue and

[2] O'Donoghue and Rabin (2015) review the present bias. Ameriks et al. (2007) measures the selc control problem by experiments and Cohen et al. (2020) review research that measures time preferences.

Rabin 1999). If this would emerge in stock market, investors may react differently to positive and negative signals. O'donoghue and Rabin (2001) point out that a subjective perception of multi-selves problem influences investors' current behavior. Therefore, if the choices and ctions of investors at each time are not consistent, there would be arbitrage opportunites in stock market.

This paper fill in this gap in the DE literature. We build dinamically consistent DE and compare the with an inconsistent model which can be seen as an one-shot deviation model from rational expectation. We consier how agents' beliefs are updated based on own past distorted beliefs. In such setting, we avoid the situations where agents forget their distorted beliefs every period and the issues related to multi-selves.

The most related paper to this analysis is Bianchi et al. (2024). They introduce the uncetatinty effect into DE. Bordalo et al. (2018) and Bordalo et al. (2019) assume that subjective uncetatinty is stationary and not revised to focus on the impact of psychological heuristics on expected values. Bianchi et al. (2024) propose the smooth DE in which subjective volatility is also revised to examine the uncetatinty effects. In contrast, this paper analyze the distorted belief updation based on distorted beliefs. This paper focus on the dynamic consistency and it allows us to dervie the condition that beliets is stationary.

Notice that DE is also related to selective memory. When decision maker observes new information, states or events whose probability increases the most come to mind first and they are oversampled in mind because of representativeness heuristics. In selective memory literature, people recall the memory from their database and the order of recalling or weights is often decreasing in history. While standard setting assumes that observing events or information are sources of current beliefs (Bordalo et al. 2020a; Nagel and Xu 2022), our results suggest that own past beliefs can be candidates of them.

3 Model: Diagnostic expectation

Assume that the firm's fundamental follows the AR(1) process

$$f_t = af_{t-1} + \eta_t \tag{1}$$

where $a \in [0, 1]$ and $\eta_t \sim N(0, \sigma_\eta^2)$ is an iid normally distributed shock. It is assumed that investors cannot observe f_t directly, instead, they observe the signal x_t which is given by

$$x_t = bx_{t-1} + f_t + \epsilon_t \tag{2}$$

where $b \in [0, 1]$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ is an iid normally distributed shock. Imposing $b \leq a$ ensures the stationarity. Bordalo et al. (2019) interpret this signal as the natural logarithm of the EPS of firm.

Rational investors update their beliefs according to the Kalman fiter to infer the current fundamental based on the information set after observing signal x_t. The expected value of f_t is updated by following.

$$E[f_t \mid x_t] = \hat{f}_t = a\hat{f}_{t-1} + K(x_t - bx_{t-1} - a\hat{f}_{t-1}) \tag{3}$$

where $K \equiv (a^2\sigma_f^2 + \sigma_\eta^2)/(a^2\sigma_f^2 + \sigma_\eta^2 + \sigma_\epsilon^2)$ is the signal-to-noise ratio, or called Kalman gain .[3] This indicates the extent to which signal fluctuations are from fundamental.

DE is a model in which agents' beliefs are distorted by the representativeness heuristic. Represen heuristic is human's tendency that they overestimate the probability of events which is a representative or typical of a parent class (Kahneman and Tversky 1972). Gennaioli and Shleifer (2010) propose the measurement of the representativeness. Based on this, Bordalo et al. (2016) assume that probability judgements are formed using the representativeness-

[3] In the steady state, the variance of fundamental is given as the solution to $a^2\sigma_f^4 + \sigma_f^2[\sigma_\eta^2 + (1-a^2)\sigma_\epsilon^2] - \sigma_\eta^2\sigma_\epsilon^2 = 0$. In our model, subjective variance of fundamental is assumed to be in stationary, which is same for previous literature.

distorted density

$$h^{\theta}(T = \tau \mid G) = h(T = \tau \mid G) \left(\frac{h(T = \tau \mid G)}{h(T = \tau \mid -G)} \right)^{\theta} Z \tag{4}$$

where $h(T = \tau|G)$ is a distribution of a variable T in a group G, $-G$ is a comparision group, $\theta \geq 0$, and Z is a constant ensuring that the distorted density $h^{\theta}(T = \tau \mid G)$ integrates to 1. θ controls the impact of the representativeness. As θ increases, the distortion of the probability increases; when $\theta = 0$, the distribution is not distorted from the true distribution $h(T = \tau \mid G)$ and it captures the rational benchmark.

Bordalo et al. (2019) apply this distorted probability judgements and propose the DE. They use the rational expectations ($N(\hat{f}_t, \sigma_f^2)$) after observing the signal x_t as a target distribution ($h(T = \tau \mid G)$) and the rational expectation ($N(a\hat{f}_{t-1}, \sigma_f^2)$) when the signal surprise is zero ($x_t = bx_{t-1} + a\hat{f}_{t-1}$) as a comparison distribution ($h(T = \tau \mid -G)$). Substituting each distribution into equation (4), DE is formed by the distorted Kalman filter.

$$E^{\theta}[f_t \mid x_t] = \hat{f}_t^{\theta} = a\hat{f}_{t-1} + K(1+\theta)(x_t - bx_{t-1} - a\hat{f}_{t-1}) \tag{5}$$

Under the DE, investors compare the current conditional rational distribution and past one and overestimate(underestimate) the probability of events which is more(less) likely to occur after observing new information. Reference distribution is a past conditional rational distribution so that past distorted beliefs (past DE) are irrelevant with current beliefs. Current DE does not have consistenty. From this perspective, we can interpret DE as one-shot deviation strategy from the rational expectations.

This dynamic inconsistency expose the DE to the critiques of multi-selves problems. In the case of multi-selves, outcome evaluations and prefferences would change by time, making it difficult to use revealed preferences and compare the data dynamically. Welfare analysis becomes also hard.

To address these issues and apply DE in various analyses, we use past distorted beliefs as reference distribution and consider the distorted belief updation based on distorted beliefs. We analyze the DE which has dynamic consistency.

Suppose that past belief is not rational.

$$E[f_{t-1} \mid x_{t-1}] = \hat{f}^d_{t-1} \tag{6}$$

The forecast error of signals and its expected value based on this distorted past beliefs are followings.

$$v^d_t = x_t - E^r[x \mid x_{t-1}]$$
$$E^r[v_t \mid x_{t-1}] = E^r[x_t \mid x_{t-1}] - E^r[E^r[x_t \mid x_{t-1}] \mid x_{t-1}] = 0$$

where $E^r[\cdot]$ is an expectation operator based on equation (6). If investors' prior beliefs are given by equation (6) and they update their beliefs by Kalman filter, their posterior beliefs are

$$E^r[f_t \mid x_t] = a\hat{f}^d_{t-1} + K(x_t - bx_{t-1} - a\hat{f}^d_{t-1}) \equiv \hat{f}^r_t \tag{7}$$

Next, we consider the DE when the prior beliefs are given by (6) equation. Distorted density shown in (4) equation requires target distribution and reference distribution. We use the current conditional distribution ($N(\hat{f}^r_t, \sigma^2_f)$) as a target distribution. For a reference distribution, we use the distribution ($N(a\hat{f}^d_{t-1}, \sigma^2_f)$) with prior beliefs of (4) equation and zero signal surprise. We denote $\hat{f}^d_t$ as the expected value of fundamental conditional on current signal with dynamic consisntency.

Proposition 1 (Dynamically Consistent Diagnostic Expectation)**.** *Under the dynamically consistent diagnostic expectation, the expected value of fundamental is updated according to following.*

$$\hat{f}^d_t = a\hat{f}^d_{t-1} + (1+\theta)K(x_t - bx_{t-1} - a\hat{f}^d_{t-1}) \tag{8}$$

Proof is shown in Appendix A.

Table 1 shows the current beliefs about fundamental based on prior belief and updation method. Top-left corresponds to the rational expectation where prior beliefs are rational and investors update their beliefs by Kalman filter. Posterior belief in this area is obtained by equation (3). Top-right corresponds to the DE proposed by Bordalo et al. (2019) shown by equation (5). Since they assume that prior beliefs are rational, they compare the updation rule and analyze the impact of distorted updation.

In contrast, in the lower row, prior beliefs are not rational. In the bottom-left, investors update their beliefs by Kalman filter without correcting prior beliefs, shown by equation (7). Bottom-right corresponds to the DE where prior beliefs are distorted and investors update their beliefs by distorted Kalman filter characterized by equation (8).

Compared to top-right one-shot DE, bottom-right DE is dynamically consistent. The signal surprise for this consistent diagnostic investors is $x_t - (bx_{t-1} + a\hat{f}^d_{t-1})$. If surprise in previous period was positive and $\hat{f}^d_{t-1}$ increased, current surprise is more likley to be negative. In addition, this surprise is incorporated to beliefs by $(1+\theta)K$ not by Kalman gain(K). Dynamically consistent DE qualitatively generates larger signal surprise and overreact to signals than rational expectation and one-shot DE.

In one-shot DE, belief overreaction is solely driven by θ, which shows the impact of the representativeness. On the contrary, in the consistent DE, overreaction can be driven by θ and inflated surprises. it implies that estimated θ under one-shot DE may be overestimated.

To consider more general situation, suppose that reference beliefs is not the conditional distribution at $t-1$ but $t-s$. In such setting, one-shot and consistent DEs are followings.

$$\hat{f}^\theta_t = a\hat{f}_{t-s} + (1+\theta)K(x_t - bx_{t-s} - a^s\hat{f}_{t-s}) \tag{9}$$

$$\hat{f}^d_t = a\hat{f}^d_{t-s} + (1+\theta)K(x_t - bx_{t-s} - a^s\hat{f}^d_{t-s}) \tag{10}$$

One-shot DE is a model which always deviates from rational expectation. Since equation (3) does not diverge, equation (9) does not diverge either. In contrast, rearranging equation (10), we obtain $\hat{f}^d_t = [1-(1+\theta)K]a\hat{f}^d_{t-s} - (1+\theta)K(x_t - bx_{t-s})$. The second term is driven by equation (2), so the first term is a key whether equation (10) diverges.

Corollary 2. *The mean value of dynamically consistent diagnostic expectation whose refer-*

ence distribution is $t-s$ conditional distribution evolves according to (10) equation. In order to be stationary process, followings must be satisfied.

$$1+\frac{1}{a^s} > (1+\theta)K \tag{11}$$

$$(1+\theta)K > 1-\frac{1}{a^s} \tag{12}$$

Since $a \in [0,1], \theta \geq 0, K \in [0,1]$, we have $(1+\theta)K \geq 0 \geq 1-1/a^s$ so that condition (12) always holds. Therefore, condition (11) determines the statonarity. If this holds, it implies that there would be upper bound for distorted Kalman gain($(1+\theta)K$), or the strength of representativeness(θ). The more persistent the fundamental process is (the higher a is), the lower this upperbound is and the less distortion for DE.

4 Simulation

We estimate the parameters of consistent DE. We compare the estimated parameters between one-shot and consistent DEs.

We follow the simulation procedure of Bordalo et al. (2019) and estimate the diagnostic parameter(θ) and other macroeconomic paramters. They use the natural logarithm of EPS as a sigmal(x_t). As expectation data, We use data on analysts' expectations from Refinitiv Eikon. We obtain mean analysts' forecasts for earnings per share(EPS) from 1982 to 2023. Sample firms are listed firms on major U.S. stock exchange (NYSE, AMEX, and NASDAQ).

Bordalo et al. (2019) obtain data of analysts' forecasts from IBES and their sample period is between 1981 and 2016. To check the diffrences of data, we firstly run the test of Coibion and Gorodnichenko (2015) (CG test) to confirm that our analysts' expectations also exhibit the overreaction tendency.

$$x_{t+h} - x_t - LTG_{t,h} = \alpha + \gamma(LTG_{t,h} - LTG_{t-k,h}) + e_{t+h} \tag{13}$$

where $LTG_{t,h}$ is an expected growth($E[x_{t+h} - x_t \mid x_t]$) of signal(EPS) for $t+h$ periods. If investors overreact to surprises, forecast revision ($LTG_{t,h} - LTG_{t-k,h}$) becomes excessive.

As a result, forecast errors ($x_{t+h} - x_t - LTG_{t,h}$) are negative on average. Therefore, if γ is negative, investors overreact to signals.

Table 2 shows the number of samples for CG test based on forecast error length(h) and forecast revision length(k). Both h and k are year level. $(k, h) = (1, 1)$ has the most samples and the sample numbers decrease in h. In particular, the sample size sharply decreases for longer forecast length($h > 3$).

Table 3 shows the result of CG test. All combinations of (k, h) have negative γ and they are statistically significant. In particular, estimated coefficients for $h = 1$ are larger negative values. Compared with Bordalo et al. (2019), our results share the sign of coefficients and the magnitude is bigger than them. We observe that analysts in our data also exhibit overreactions to signals.

Next, we rum SMM to estimate parameters $(a, b, \sigma_\eta, \sigma_\epsilon, \theta, s)$. $(a, b, \sigma_\eta, \sigma_\epsilon)$ are macroeconomic parameters which govern fundamental and signal. θ is a parameter of the strength of the representativeness. s is a sluggishness parameter which indicates the quarters of reference distribution.

We make parameter combinations on a grid defined by $a, b \in [0, 1]$, $\sigma_\eta, \sigma_\epsilon \in [0, 0.5], \theta \in [0, 2]$,and $s \in \{1, \cdots, 20\}$ quarters. Step is 0.025 for a, 0.1 for b, 0.05 for σ_η and σ_ϵ, 0.1 for θ.

Firstly, we simulate a time series of fundamental and signal. We compute the asociated one-shot DE($\hat{f}_t^\theta$) and consistent DE($\hat{f}_t^d$). Then, we calculate the long-term growth of signal under both DEs.

$$LTG_{t,h}^\theta = E^\theta[x_{t+h} - x_t] = -(1 - b^h)x_t + a^h \frac{1 - (b/a)^h}{1 - (b/a)} \hat{f}_t^\theta \tag{14}$$

$$LTG_{t,h}^d = E^d[x_{t+h} - x_t] = -(1 - b^h)x_t + a^h \frac{1 - (b/a)^h}{1 - (b/a)} \hat{f}_t^d \tag{15}$$

where $E^\theta[\cdot]$ and $E^d[\cdot]$ is an expectation operator of one-shot and consistent DEs.

Using this $LTG_{t,h}$, we compute the forecast errors and forecast revision to run CG test. Following Bordalo et al. (2019), we use $(k, h) = (1, 4)$ and $(3, 4)$. We denote $\hat{\gamma}_1$ and $\hat{\gamma}_3$ for each regression coefficient. In addition, we compute the autocorrelation $\hat{\rho}_l = cov(x_t, x_{t-l})/var(x_t)$

at lags $l = 1$ through 4 years. This yileds the coefficient vector.

$$v(a, b, \sigma_\eta, \sigma_\epsilon, \theta, s) = (\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_4, \hat{\gamma}_1, \hat{\gamma}_3)$$

We repeat this exercise for all parameter combinations. At the end, we estimate the parameters by picking the combination that minimizes the Euclidean distance loss function

$$l(v) = \|v - \bar{v}\|$$

where $\bar{v}$ is the coefficient vector estimated from the realized data and we call this target vector. It is shown by table 4.

We run this simulation for 30 independent times and obtain 30 combinations of estimated parameters. Using these combinations, we analyze the differences of one-shot and consistent DEs.

5 Results

5.1 Baseline simulations

Firstly, we focus on the simulation of one-shot DE, which is a replication of Bordalo et al. (2019). Table 5 shows the mean and standrard deviation of 30 estimated parameters. (a) is a result of one-shot DE. All parameters are statistically significantly different from zero. We find that fundamental is strongly persistent, but b is 0.3083, implying that signal is not persitent. Looking at the variance of error terms, we observe that σ_ϵ is larger than σ_η. It implies that signal contains large noises and it is less informative. Compared with Bordalo et al. (2019), the simulation in this paper has higher values of a and lower value of b. They also observe that σ_η is larger than σ_ϵ, suggesting that signal is informative.

In contrast to macroeconomic parameters, for the most interested parameter of the representativeness heuristics, our result is close to previous research. The mean value of θ is 1.0683, which is slightly higher than previous researches. Because signal surprises are incorporated into investors' belief by $(1 + \theta)K$, if $\theta = 1$, posterior beliefs incorporate signal surpise twice more than rational expectations. Because $\theta = 0$ is also included to the parameter combinations but not selected as best value, investors are likely to overreact to signals according to DE. Notice that sluggishness which indicates the periods of reference distribution is 6.7667 quarters in this simulation, which is lower than previous research.

Next, we focus on the simulation for dynamically consistent DE. Table 5 (b) shows that fundamental is still persistent, but mean value of b is 0.4233, implying that signal is not persistent as well. In terms of variance of error terms, σ_ϵ is larger than σ_η, but the diffrence of these variances is smaller in consistent model than one-shot model, implying that consistent diagnostic model has relatively more informative than one-shot model.

5 (c) shows the difference of both model with standard errors. We look at the θ and sluggishness which are related to DE. We find that consistent model has lower values than one-shot model. It is statistically significant at 5% level for θ. It implies that consistent model generates lower overreaction than one-shot model, which is consistent with corollary 2.

5.2 Robustness test

Baseline simulations include $\theta = 0$ as parameter combinations and estimate the θ which minimizes the Euclidean loss function. For one-shot and consistent DE models, $\theta = 0$ corresponds to rational expectations, but our results show that both diagnostic models fit better than rational expectation model. We run another simulation to confirm that the consistent model has lower value of θ than one-shot model.

We test the null hypothesis that the one-shot model is correct. In this simulation, we fix $(a, b, \sigma_\eta, \sigma_\epsilon)$ to the estimated ones for one-shot model: $(a, b, \sigma_\eta, \sigma_\epsilon) = (0.99, 0.3083, 0.165, 0.3537)$. Based on these macroeconomic parameters, we estimate the best combinations of (θ, s). Fixing the macroeconomic parameters allows us to directly examine the (θ, s) in both DEs.

Following the baseline simulation, we simulate the time series of fundamental and signal and calculate the associated expected growth by one-shot and consistent model. Then, we run CG test and make the coefficient vector $(\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_4, \hat{\gamma}_1, \hat{\gamma}_3)$. At the end, we estimate the parameters by picking the combination that minimizes the Euclidean distance loss function. We run 360 independent simulations.

Figure 1 shows the Euclidean distance based on combinations of θ and sluggishness. (a) is one-shot model and (b) is consistent model. We observe that there are no valid distance for large sluggishness in consistent model. When reference period is large, the magnitude of signal surprises is more likely to be large. Consistent DE reacts to these signals by distorted Kalman gain $(1+\theta)K$ so that current expected values of fundamental and expected growth of signal have large values. In addition, signal surprises are determined by this inflated expected values so that surprises are systematically large in the future. Repeating this makes expected values reach infinity eventually, in which the Euclidean distance cannot be calculated.

Even the reference period is short, a similar problem would occur if θ is large. Although initial surpise is small, distorted Kalman gain incorporate this into beliefs and make expected value of fundamental inflated. Subsequent surprises gradually become large and make expected value infinite.

Figure 2 shows the relation between θ and Euclidean distance around the estimated sluggishness. Reference period for (a) is 6 quarters and one for (b) is 7 quarters. They are aound estimated sluggishness in table 5 (a). We find that consistent models have no Euclidean distance when θ is large, which is same to Figure 1. We also observe that Euclidean distance is not linear in θ and it increases when θ is over 0.5 for both model.

The Euclidean distance is at minimum around $\theta = 0.4$ for one-shot model. This value is different from the result in table 5 (a). One of the potential explanations is the difference of macroeconomic parameters $(a, b, \sigma_\eta, \sigma_\epsilon)$. Table 5 are derived from combinations of all parameters whereas macroeconomic parameters are fixed in this simulation. Fixed parameters are not used in previous simulations. Therefore, there are no guarantee that estimated θ conditional on macroeconomic parameters is same to previous one.

For the consistent model, Euclidean distance is minimum at $\theta = 0.1$ and increases in θ. In addition, expected value of fundamental diverges if θ is over 0.5 and no Euclidean distances are obtained.

Comparing both model, their distances are same at $\theta = 0$ because both models corresponds to rational expectation model at that point. We also observe that consistent model has lower minimum distance than one-shot model. It implies that consistent model fits better to realized data.

6 Conclusion

Under DE, investors overestimate the representative states and overreact to observed signals. Their beliefs are distorted because of the representativeness heuristics.

Most literature assumes that agents compare the current rational beliefs and past one and overestimate the representative states of current beliefs. However, this setting makes current distorted beliefs irrelevant with past distorted beliefs and causes the multi-selves problem.

We build the dynamically consistent DE in which agents update their beliefs based on their past distorted beliefs. We find that there is an upper bound for the strength of the representativeness to prevent DE from diverging infinity.

We run SMM to estimate the parameters of the representativeness and the reference periods. We find that consistent model has lower representativeness impacts and refers to more recent history, which are consistent with our model. We also observe that consistent model has lower Euclidean distances than one-shot model if we fix the macroeconomic parameters.

Our main contribution is offering the dynamically consistent DE model. We offer necessary conditions for DE to be stationary, and provide the evidence that one-shot model overestimates the strength of the representativeness.

This paper also has implications about analysts' rationality. Although analysts' overreaction is observed by many studies(La Porta 1996, Gennaioli and Shleifer 2018) which is also observed in our data, our results suggest that analysts are somehow consistent with their beliefs. We offer new insights fr analysts' characteristics.

Data difference is a caveat of this paper. We obtained data from Refinitiv Eikon and the sample periods are between 1982 to 2023. Bordalo et al. (2019) obtain data from IBES between 1981 and 2016. These differences may be one reasons of the etimated macroeconomic parameters shown in table 5. Compared with prior literature, our results are characterized with high persitence of fundamental, low persitence of signal, and less informativeness of signal. Although the most important parameters are in line with prior literature, these differences may influence our estimated values.

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A Proof of proposition 1

Proof. Probability density of DE is given by equation (4). For dynamically consistent DE, target distribution is a conditional distribution ($N(\hat{f}_t^r, \sigma_f^2)$) after observing current signal and reference distribution is a conditional distribution ($N(a\hat{f}_{t-1}^d, \sigma_f^2)$) when the signal surprise is zero. Prior belief of both distribution is given by equation (6). We substitute these distribution into (4) equation.

$$h^d(f_t \mid x_t) = \frac{1}{\sqrt{2\pi\sigma_f^2}} \exp\left(-\frac{(f_t - \hat{f}_t^r)^2}{2\sigma_f^2}\right) \left[\frac{\frac{1}{\sqrt{2\pi\sigma_f^2}} \exp\left(-\frac{(f_t - \hat{f}_t^r)^2}{2\sigma_f^2}\right)}{\frac{1}{\sqrt{2\pi\sigma_f^2}} \exp\left(-\frac{(f_t - a\hat{f}_{t-1}^d)^2}{2\sigma_f^2}\right)}\right]^\theta Z \tag{A.1}$$

where Z is a constant eunsuring that the distorted density $h^d(f_t \mid x_t)$ integrates to 1. Summarizing this, we find that

$$h^d(f_t \mid x_t) = \frac{1}{\sqrt{2\pi\sigma_f^2}} \exp\left(-\frac{(f_t - \hat{f}_t^r)^2 + \theta[(f_t - \hat{f}_t^r)^2 - (f_t - a\hat{f}_{t-1}^d)^2]}{2\sigma_f^2}\right) Z \tag{A.2}$$

Since $\hat{f}_t^r$ and $\hat{f}_{t-1}^d$ are constant, the exponent term is

$$-\frac{(f_t - [\hat{f}_t^r + \theta(\hat{f}_t^r - \hat{f}_{t-1}^d)])^2 + C}{2\sigma_f^2} \tag{A.3}$$

where $C = (\hat{f}_t^r)^2 + \theta[(\hat{f}_t^r)^2 - (a\hat{f}_{t-1}^d)^2] - [\hat{f}_t^r + \theta(\hat{f}_t^r - a\hat{f}_{t-1}^d)]^2$ is constant and does not depend on f_t. Since $h^d(f_t \mid x_t)$ is a probability density, it integrates to 1.

$$\int h^d(f_t \mid x_t) = \int \frac{1}{\sqrt{2\pi\sigma_f^2}} \exp\left(-\frac{(f_t - [\hat{f}_t^r + \theta(\hat{f}_t^r - \hat{f}_{t-1}^d)])^2}{2\sigma_f^2}\right) \exp\left(-\frac{C}{2\sigma_f^2}\right) Z df_t \tag{A.4}$$

$$= \exp\left(-\frac{C}{2\sigma_f^2}\right) Z \int \frac{1}{\sqrt{2\pi\sigma_f^2}} \exp\left(-\frac{(f_t - [\hat{f}_t^r + \theta(\hat{f}_t^r - \hat{f}_{t-1}^d)])^2}{2\sigma_f^2}\right) df_t = 1 \tag{A.5}$$

Term inside the integral is interpreted as a normal density with mean $\hat{f}_t^r - \hat{f}_{t-1}^d$ and variance

σ_f^2 so that latter term is 1.

$$\exp\left(-\frac{C}{2\sigma_f^2}\right) Z * 1 = 1 \tag{A.6}$$

$$Z = \exp\left(\frac{C}{2\sigma_f^2}\right) \tag{A.7}$$

Substituting Z into density function,

$$h^d(f_t \mid x_t) = \frac{1}{\sqrt{2\pi\sigma_f^2}} \exp\left(-\frac{(f_t - [\hat{f}_t^r + \theta(\hat{f}_t^r - \hat{f}_{t-1}^d)])^2}{2\sigma_f^2}\right) \tag{A.8}$$

Dynamically consistent DE is characterized by mean $\hat{f}_t^r + \theta(\hat{f}_t^r - \hat{f}_{t-1}^d)$ and variance σ_f^2. Because $\hat{f}_t^r$ is given by equation (7), we have

$$\hat{f}_t^d = \hat{f}_t^r + \theta(\hat{f}_t^r - a\hat{f}_{t-1}^d) = a\hat{f}_{t-1}^d + (1+\theta)K(x_t - bx_{t-1} - a\hat{f}_{t-1}^d) \tag{A.9}$$

□

Table 1. Beliefs based on prior beliefs and updation type.

prior\update	Rational	Diagnostic
Rational ($\hat{f}_{t-1}$)	[rational expectation] $\hat{f}_t = a\hat{f}_{t-1} + K(x_t - bx_{t-1} - a\hat{f}_{t-1})$	[one-shot Diagnostic expectation] $\hat{f}_t^{\theta} = a\hat{f}_{t-1} + (1+\theta)K(x_t - bx_{t-1} - a\hat{f}_{t-1})$
Distorted ($\hat{f}_{t-1}^d$)	$\hat{f}_t^r = a\hat{f}_{t-1}^d + K(x_t - bx_{t-1} - a\hat{f}_{t-1}^d)$	[consistent Diagnostic expectation] $\hat{f}_t^d = a\hat{f}_{t-1}^d + (1+\theta)K(x_t - bx_{t-1} - a\hat{f}_{t-1}^d)$

This table shows the beliefs based on prior beliefs and updation type. Rows are prior beliefs and columns are updation type.

Table 2. Sample numbers for Coibion and Gorodnichenko (2015) test.

	h=1	h=2	h=3	h=4	h=5
k=1	31480	29289	20470	7066	4624
k=2	28797	26958	18573	5900	3740
k=3	26603	25002	16971	5121	3171
k=4	24644	23238	15569	4473	2719
k=5	22831	21586	14272	3903	2317

This table shows the sample numbers used for CG tests. Row k shows the years of forecast revisions and column h shows the years of forecast length.

Table 3. Results of Coibion and Gorodnichenko (2015) test.

	h=1	h=2	h=3	h=4	h=5
k=1	-0.754***	-0.601***	-0.597***	-0.685***	-0.677***
(0.009)	(0.008)	(0.01)	(0.018)	(0.023)	
k=2	-0.781***	-0.594***	-0.62***	-0.706***	-0.678***
(0.009)	(0.008)	(0.009)	(0.018)	(0.022)	
k=3	-0.745***	-0.599***	-0.625***	-0.706***	-0.664***
(0.009)	(0.008)	(0.009)	(0.017)	(0.022)	
k=4	-0.779***	-0.597***	-0.625***	-0.671***	-0.682***
(0.009)	(0.008)	(0.01)	(0.017)	(0.024)	
k=5	-0.795***	-0.612***	-0.646***	-0.68***	-0.628***
(0.009)	(0.008)	(0.01)	(0.019)	(0.024)	

This table shows the regression coefficient of forecast errors on forecast revision. Row k shows the years of forecast revisions and column h shows the years of forecast length.

Table 4. Target coefficient values.

$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{\gamma}_1$	$\hat{\gamma}_3$
0.8947	0.8225	0.802	0.7708	-0.685	-0.706

ρ_l is a autocorrelation of pooled EPS for $l = 1, 2, 3, 4$ years. γ_h is a regression coefficient of CG test for $h = 1, 3$ years. These are target vectors for simulations.

Table 5. Estimated parameters.

(a) One-shot diagnostic expectation.

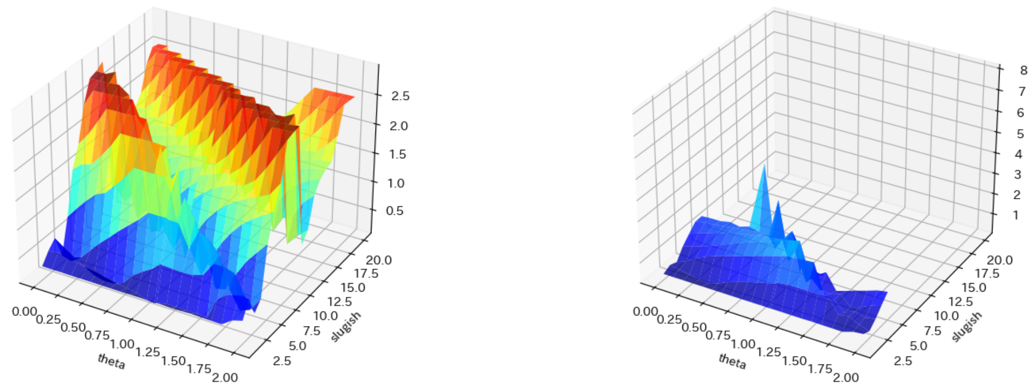
	a	b	σ_η	σ_ϵ	θ	s
mean	0.99	0.3083	0.165	0.3537	1.0683	6.7667
std. dev	0.0124	0.1907	0.1287	0.1181	0.6133	7.2026

(b) Consistent diagnostic expectation.

	a	b	σ_η	σ_ϵ	θ	s
mean	0.9858	0.4233	0.1887	0.3037	0.805	4.55
std. dev	0.0133	0.2925	0.1384	0.1313	0.6458	5.9273

(c) Differences between consistent and one-shot model.

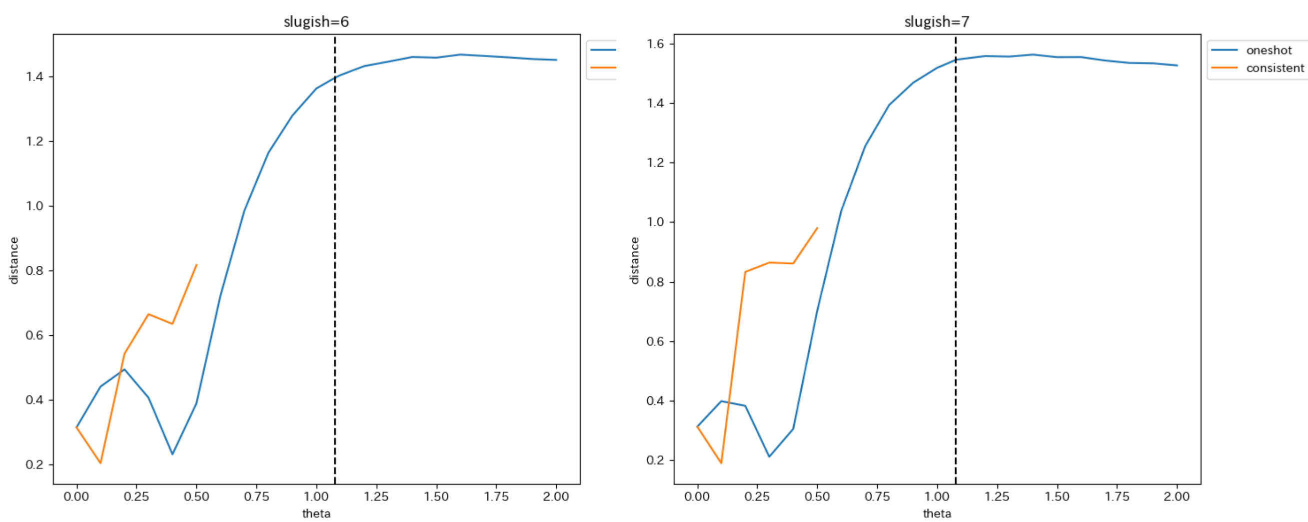
	a	b	σ_η	σ_ϵ	θ	s
mean	-0.0*	0.12*	0.02	-0.05*	-0.26*	-2.22
std. error	0.078	0.012	0.332	0.03	0.024	0.068



(a) One-shot model. (b) Consistent model.

Figure 1: Euclidean distance of (θ, s).

This figure shows the average Euclidean distance between coefficient vector calculated by combination of (θ, s) and target vector. For consistent model, no distances are recorded in high sluggish because beliefs diverge.



(a) Sluggish=6. (b) Sluggish=7.

Figure 2: Euclidean distance of (θ, s).

This figure shows the average Euclidean distance between coefficient vector calculated by combination of (θ, s) and target vector. Sluggish is fixed to 6 qurters for (a) and 7 quarter for (b).